The results of the calculations for cavities of various configurations are shown in Fig. 3. It is clear from the figures that the effective absorption coefficient of the cavity is higher for parallel than for diffuse radiation, except for a cylindrical cavity with $H < 2.0$. With increasing depth of the cavity, or a decrease in the angle of taper, ε_{eff} approaches a certain limiting value asymptotically ($\varepsilon_{eff} = 1$ for parallel radiation, and $\varepsilon_{eff} = 0.943$ for diffuse radiation). Therefore, increasing H beyond 4.0 or decreasing θ below 0.5 for diffuse radiation increases ε_{eff} only slightly. The value of ε_{eff} is more effectively increased by increasing the emissivity of the cavity walls and decreasing the radius of the cavity opening. For parallel radiation decreasing the angle of taper θ below 0.5 is also effective in increasing ε_{eff}. By choosing optimum values of all four parameters it is possible to produce a calorimeter for thermal radiation with characteristics closely approaching those of a black body.

NOTATION

θ, angle of taper of cavity; H, height of cavity; R, radius; R_0, radius of opening of cavity; ε, emissivity of cavity walls; ε_{eff}, effective emissivity of cavity; Q_{in}, incident heat; Q_{ref}, reflected heat; λ, reflection coefficient of cavity walls.

LITERATURE CITED

1. R. Siegel and J. Howell, Thermal Radiation Heat Transfer, McGraw-Hill, New York (1972).
2. E. M. Sparrow and A. Haji-Sheikh, "A generalized variational method for calculating radiant interchange between surfaces," J. Heat Trans., Trans. ASME, Sec. C, 87, 103 (1965).
3. M. N. Otsisik, Complex Heat Exchange [in Russian], Mir, Moscow (1976).
4. D. McCracken and W. Dorn, Numerical Methods and FORTRAN Programming, Wiley, New York (1964).

SOME FEATURES OF THE THERMALLY CONCENTRATED CONVECTIVE MOTION OF A HARDENING BINARY MELT AND THE IMPURITY DISTRIBUTION

P. F. Zavgorodnii

UDC 621.746.7.001

Some features of the thermally concentrated convective motion of a binary melt, hardening in a closed rectangular region with movable boundaries, and the impurity distribution are investigated numerically.

It was shown in [1] that the impurity distribution in the hardening part of a crystallizing fixed melt is mainly determined by the nature of the change in the impurity concentration at the boundary between the hard and liquid phases. It was established in [2] that convective mixing of the liquid nucleus due to its temperature nonuniformity has a considerable effect on the nature of the impurity distribution at the phase-transition boundary and, consequently, on the impurity distribution in the hardening part of the crystallizing melt.

However, some features of the hardening of a binary melt were ignored in [1, 2]. Thus, when a binary melt hardens a concentrational nonuniformity develops in the liquid nucleus together with a temperature nonuniformity, due to the difference in the solubility of the impurity in the solid and liquid phases. The result of the combined action of the temperature and concentration nonuniformities will be the occurrence and development of a thermally concentrated gravitational convective motion in the liquid nucleus of the hardening alloy, the features of which should also manifest themselves in the nature of the impurity distribution.

Consider a rectangular region filled with melt with initial temperature $T_0 > T_K$ and an initial impurity content c_0, with relative dimensions $l_1 = L_1/x_0$, $l_2 = L_2/x_0$. The region in which the melt exists is situated in space such that $0 \leq x_1 \leq L_1$, $0 \leq x_2 \leq L_2$, and the direction of the acceleration due to gravity determines the positive di-

---

Donets State University.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 1, pp. 118-125, July, 1980. Original article submitted April 2, 1979.

0022-0841/80/3901-0797$07.50

rection of the Ox_2 axis. The hardening boundary is assumed to be plane, and directly separates the solid and liquid phases.

At a certain instant of time ($t > 0$), when the temperature of the boundaries of the region falls abruptly to the crystallization temperature of the melt, a solid phase begins to be formed, where the variations with time of the thickness of the solid phase and liquid nucleus are assumed to be known functions [3]. If we take as the characteristic dimensions the width $L_1(x_0 = L_1)$ of the region considered, then they have the following form in dimensionless notation:

$$R_1 = R_2 = \alpha\sqrt{\mathrm{Fo}},\ \varepsilon_1 = 1 - \alpha\sqrt{\mathrm{Fo}},\ \varepsilon_2 = l_2 - \alpha\sqrt{\mathrm{Fo}}.$$

This choice is quite justified. It follows from [4] that the solution of this kind of problem, taking into account the effect of convective mixing of the liquid nucleus due to heat transfer through the solid phase, which essentially determines the position with time of the phase-transition boundary, does not introduce any considerable changes into the final result obtained, but considerably complicates the program and increases the time taken to carry out the calculation.

We will choose as the object being investigated a melt of low-carbon steel, the physical properties of which are given in [5].

The mathematical model of the process consists of the following equations taken in dimensionless form: the equation of momentum transfer in the Boussinesf approximation with the condition that the characteristic velocity u_0 and the characteristic pressure difference $P_{max} - P_{min}$ are given by the expressions

$$u_0 = \frac{D}{x_0},\quad P_{\max} - P_{\min} = \rho\frac{D^2}{x_0^2},$$

$$\frac{\partial \bar{U}}{\partial \mathrm{Fo}} + (\bar{U}\nabla)\,\bar{U} = -\nabla\pi + \mathrm{Sm}\Delta\,\bar{U} - \bar{e}_2\mathrm{Sm}^2\mathrm{Gr}\Theta - \bar{e}_2\mathrm{Sm}^2\mathrm{Gr}_D(S-1); \tag{1}$$

The heat-transfer equation

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} + (\bar{U}\nabla)\,\Theta = \frac{1}{\mathrm{Lu}}\Delta\Theta; \tag{2}$$

the mass-transfer equation

$$\frac{\partial S}{\partial \mathrm{Fo}} + (\bar{U}\nabla)\,S = \Delta S; \tag{3}$$

and the equation of continuity

$$\nabla\bar{U} = 0.$$

In this case the range of variation of the variables η_1 and η_2 are as follows:

$$R_1 \leqslant \eta_1 \leqslant \varepsilon_1 \text{ and } R_2 \leqslant \eta_2 \leqslant \varepsilon_2.$$

In order to obtain a unique solution of Eqs. (1)-(4) we will add the following initial conditions

$$\mathrm{Fo} = 0:\ \bar{U} = 0,\ \Theta = 1,\ S = 1$$

and the boundary conditions

$$\eta_1 = R_1:\ v_2 = 0,\ \Theta = 0,\ -\frac{\partial S}{\partial \eta_1} = R_1'(1-k)S,$$

$$\eta_1 = \varepsilon_1:\ v_2 = 0,\ \Theta = 0,\ -\frac{\partial S}{\partial \eta_1} = \varepsilon_1'(1-k)S,$$

$$\eta_2 = R_2:\ v_1 = 0,\ \Theta = 0,\ -\frac{\partial S}{\partial \eta_2} = R_1'(1-k)S,$$

$$\eta_2 = \varepsilon_2:\ v_1 = 0,\ \Theta = 0,\ -\frac{\partial S}{\partial \eta_2} = \varepsilon_2'(1-k)S.$$

We will introduce the current function

$$v_1 = \frac{\partial \psi}{\partial \eta_2}, \quad v_2 = -\frac{\partial \psi}{\partial \eta_1},$$

which identically satisfies (4), the velocity vortex $\varphi = \operatorname{curl} U$, and we will also change, using the new variables

$$\zeta_1 = \frac{\eta_1 - R_1}{\varepsilon_1 - R_1}, \quad \zeta_2 = \frac{\eta_2 - R_2}{\varepsilon_2 - R_2}$$

from the region of rectangular cross section to the region of a unit square, so that $0 \leq \zeta_2 \leq 1$ and $0 \leq \zeta_1 \leq 1$ throughout the hardening process [2, 6].

To solve this problem numerically we used the finite-difference method of alternating directions (the longitudinal-transverse scheme) [7], in which the equations are split with respect to the coordinates ζ_1 and ζ_2, simultaneously using the method of fractional time steps [8].

By using the integro-interpolation method and determining the running coefficients [7] the system of equations and boundary conditions were reduced to a system of algebraic equations which were solved on the Dnepr-21 computer [9].

The general character of the thermally concentrated gravitational convective flow of the liquid nucleus of a hardening alloy and the nature of the impurity distribution was investigated numerically with $Gr = Gr_D = 0.2 \cdot 10^7$, $l_2 = 3$, $\alpha = 10$,

In addition, we also investigated numerically the effect of the ratio of the Grashof hydrodynamic and diffusion numbers, for $Gr = 0.2 \times 10^7$, $Gr_D = 0.2 \times 10^8$ and $Gr = 0.2 \times 10^8$ and $Gr_D = 0.2 \times 10^7$, on the nature of the thermally concentrated gravitational convective motion for $l_2 = 3$ and $\alpha = 10$, and also the effect of a change in the relative height of the crystallizer cavity in the range of values $l_2 = 1$, 2, and 3 for $Gr = Gr_D = 0.2 \times 10^7$, $\alpha = 10$. The equilibrium impurity distribution coefficient in all cases was assumed to be $k = 0.5$ (the initial concentration of carbon in the iron $c_0 \leq 0.3\%$).

From the condition for mathematical stability and fairly high accuracy by means of fractional calculations of the spatial grid we determined the dimension to be 32×32.

Analysis of the results of the calculation enable us to draw the following conclusion. In a hardening binary melt the combined action of the temperature and concentration nonuniformities lead to the occurrence and the development in the liquid nucleus of a hardening melt of thermally concentrated gravitational convective motion. The convective motion can be divided into three periods (Fig. 1, curve 1). The first is the period of acceleration of the melt to the first extremal value of the velocity, corresponding to the greatest of its maximum values. During this period the convective motion of the melt is determined by the temperature nonuniformity of the liquid nucleus, and hence, the nature of the convective motion at this stage is identical with the nature of the thermal-gravitational convective motion, which has the form of two closed vortices, symmetrically placed with respect to the vertical axis of the cavity of the crystallizer. In this case, along the vertical boundaries of the phase transition the melt drops to the bottom part, and in the central region it moves in the direction of the leading part of the crystallizer cavity (Fig. 2a).

The second period is a transition period. Its duration is determined by the time interval between the first and third extremal values of the velocity. During the second period there is a readjustment of the velocity field due to the reduction in the effect of the temperature nonuniformity and an increase in the effect of the concentration nonuniformity on the hydrodynamics of the liquid nucleus. This leads to degeneration of the vortex of the convective motion due to the temperature nonuniformity, and to the occurrence and development of a vortex of convective motion, due to the concentration nonuniformity, and is in a direction opposite to the initial one (Fig. 2b).

The third period is the period during which the concentration nonuniformity has a decisive effect on the hydrodynamics of the liquid nucleus. The convective motion during the third period, as in the first period, has the form of two closed vortices situated symmetrically with respect to the vertical of the crystallizer cavity, but with a direction of motion opposite to that in the first period (Fig. 2c).

For $Gr = 0.2 \times 10^7$, $Gr_D = 0.2 \times 10^8$ when the level of the intensity of the development of the convective motion is reduced, the duration of the first and second periods is reduced and the third period is increased. The value of the velocity at points of the first and second extrema of the velocity is reduced, while it increases at the point of the third extremum (Fig. 1, curve 2). This is obviously due to the increase in the effect of its con-

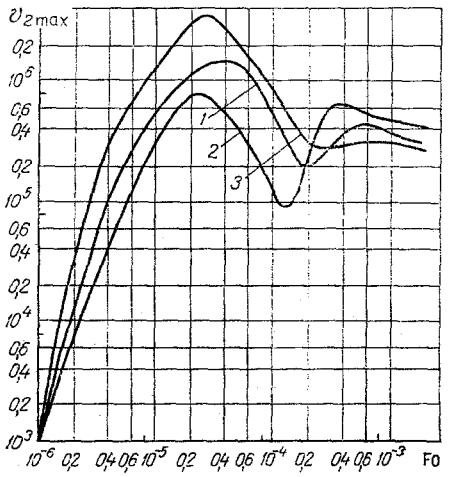

Fig. 1. Time distribution of the maximum values of the velocity of the descending flows ($l_2 = 3$, $\alpha = 10$); 1) $Gr = Gr_D = 0.2 \times 10^7$; 2) $Gr = 0.2 \times 10^7$, $Gr_D = 0.2 \times 10^8$; 3) $Gr = 0.2 \times 10^8$, $Gr_D = 0.2 \times 10^7$.

centration nonuniformity on the hydrodynamics of the liquid nucleus as a result of the increase in the Grashof diffusion number.

For $Gr = 0.2 \times 10^8$ and $Gr_D = 0.2 \times 10^7$ when the level of the intensity of the development of the convective motion is increased, the period of acceleration of the melt to the first extremal value of the velocity is reduced. Nevertheless, due to the considerable increase in the duration of the second period, its third period is reduced. The value of the velocity at points of the first and second extrema therefore increases, while it is reduced at the point of the third extremum (Fig. 1, curve 3). This can be explained by the relation between the temperature nonuniformity and the concentration nonuniformity of the liquid nucleus, which is established when the effect of the temperature nonuniformity increases due to the increase in the Grashof hydrodynamic number.

In cavities with small relative heights in the range of values $l_2 = 1, 2, 3$, the duration of the first and second periods is reduced, while the duration of the third period is increased.

In addition, it should be noted that in the period of time comprising about 2% of the total hardening time from the beginning of the process, some increase in the level of the velocity in cavities with small relative heights is observed over the level of the velocity of convective motion of the melt in cavities with large relative heights. In the next picture the distribution of the velocities changes into the opposite, and in cavities with reduced relative heights a reduction in the overall level of the velocity of convective mixing of the liquid nucleus is observed.

The explanation of this is obviously as follows.

When the relative height of the crystallizer cavity is reduced the ratio of the overall length of the phase-transition boundary to the area of the liquid nucleus increases. This facilitates an intensification of the cooling of the liquid nucleus and thereby accelerates the occurrence and development of the temperature nonuniformity, which is also responsible for the higher level of the velocity of convective motion of the melt. The increased level of the velocity of convective mixing of the liquid nucleus under more intense cooling conditions obviously facilitates the more rapid degeneration of the temperature nonuniformity, which leads to a reduction in its effect on the hydrodynamics of the liquid nucleus.

On the other hand, when the relative height of the crystallizer cavity is reduced the value of the ratio of the overall length of the phase-transition boundary to the area of the hardening melt is reduced. This leads to an increase in the impurity concentration at the phase-transition boundary, which, under conditions of increasd velocity of convective mixing of the liquid nucleus, facilitates a more rapid occurrence and development of concentration nonuniformity.

Since, for this case $\rho_n > \rho_{mp}$, the concentration nonuniformity has an effect on the hydrodynamics of the liquid nucleus facilitating a reduction in the level of the velocity of convective mixing. Nevertheless, an increase in the impurity content in the liquid nucleus due to fairly intense transfer of the impurity from the phase-transition boundary into the depth of the liquid nucleus at this stage of the development of the concentration non-

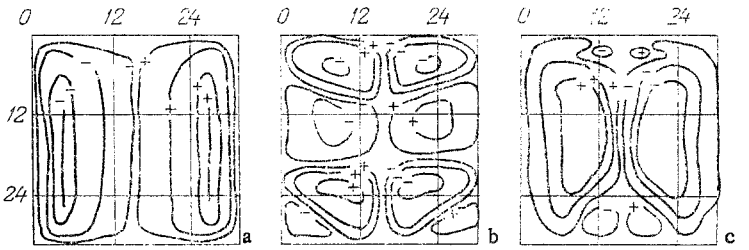

Fig. 2. Isolines of the current function ($Gr = Gr_D = 0.2 \times 10^7$, $l_2 = 3$, $\alpha = 10$); a) $Fo = 0.85 \times 10^{-5}$; b) $Fo = 0.175 \times 10^{-3}$; c) $Fo = 0.49 \times 10^{-3}$.

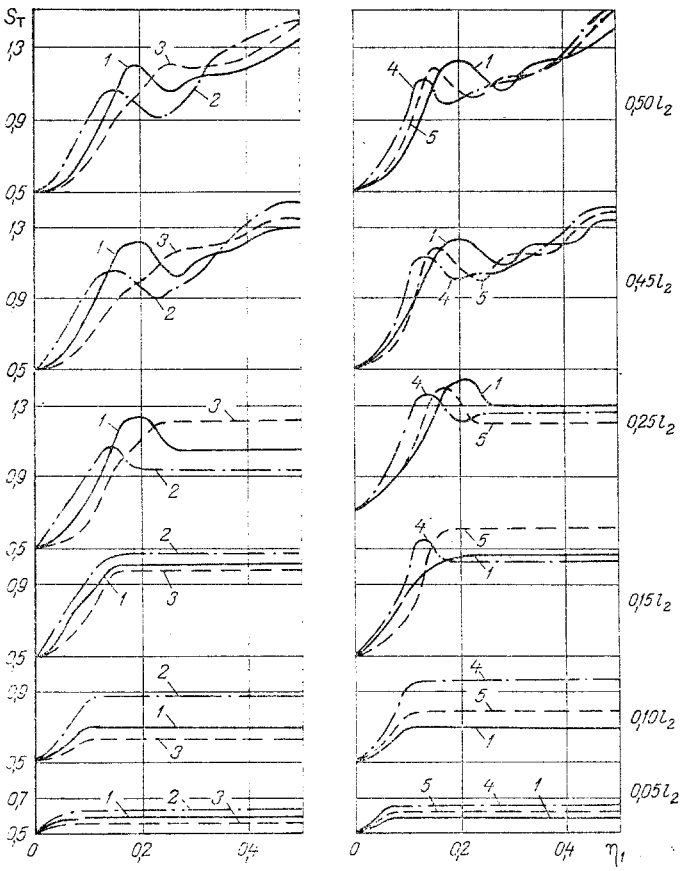

Fig. 3. Impurity distribution in a hardened alloy, $l_2 = 3$, $\alpha = 10$: 1) $Gr = Gr_D = 0.2 \times 10^7$; 2) $Gr = 0.2 \times 10^7$, $Gr_D = 0.2 \times 10^8$; 3) $Gr = 0.2 \times 10^8$, $Gr_D = 0.2 \times 10^7$; $Gr = Gr_D = 0.2 \times 10^7$, $\alpha = 10$: 4) $l_2 = 1$; 5) $l_2 = 2$.

uniformity, and subsequent reduction in the overall level of the rate of convective mixing of the liquid nucleus, produce a situation which helps to reduce the intensity of the development of the concentration nonuniformity.

Hence, the combination of the above factors in the final analysis also characterizes the above-mentioned nature of the thermally concentrated gravitation convective motion of the melt.

An analysis of the results of a numerical investigation of the nature of the impurity distribution in the melt, hardened under thermally concentrated gravitational convective motion conditions, enables us to draw the following conclusion.

The impurity distribution has a quite clear tendency to increase its content from the periphery of the hardening melt towards the central part (Fig. 3, curve 1). This is in good agreement with the results obtained in [2, 10], in which it was shown that a similar phenomenon occurs for an impurity distribution in a melt hardened both under conditions of a fixed liquid nucleus, and under conditions of thermal gravitational convective mixing.

An important feature of the impurity distribution in a melt hardened under conditions of thermally concentrated gravitational convective mixing of the liquid nucleus is the formation between the periphery and its central part of a solid-phase with extremal values of the impurity concentration (Fig. 3, curve 1).

The formation of a solid phase with extremal values of the impurity concentration is identical in time with the period of adjustment of the velocity field of the convective motion, and is obviously due to the change in the impurity concentration at the phase-transition boundary due to the change in the intensity of convective transfer from the phase-transition boundary into the depth of the liquid nucleus at the given stage of the formation of the solid phase.

For a value of the Grashof hydrodynamic and diffusion numbers $Gr = 0.7\times 10^7$ and $Gr_D = 0.2\times 10^8$, the quantitative difference between the extremal values of the impurity concentration increase, while the solid phase itself with extremal values of the impurity concentration is shifted towards the peripheral part of the hardening alloy (Fig. 3, curve 2).

For $Gr = 0.2\times 10^8$ and $Gr_D = 0.2\times 10^7$ the quantitative differences between the extremal values of the impurity concentration are reduced considerably, while the nature of the impurity distribution itself approaches the nature of the impurity distribution in the melt hardened under thermal gravitational convective mixing conditions of the liquid nucleus [2], (Fig. 3, curve 3).

As in the first case, in the second and third cases the nature of the impurity distribution is due to the features of the hydrodynamics of the liquid nucleus of the hardening melt.

In crystallizers with low values of the relative heights in the range $l_2 = 1, 2$, and 3, together with some reduction in the value of the quantitative difference between the extremal values of the impurity concentration there is also a considerable displacement of the solid-phase zone with extremal values of the impurity concentration towards the periphery of the hardening melt.

Hence, in a melt hardening under thermally concentrated gravitational convective mixing of the liquid nucleus a zone with extremal parts of impurity content is formed between the periphery and the central part of the forming solid phase, which leads to an increase in the nonuniformity of its distribution.

An increase in the ratio of the Grashof hydrodynamic and diffusion numbers leads to degeneracy of the zone with extremal values of the impurity concentration and hence facilitates an increase in the uniformity of its distribution.

In crystallizers with lower values of the relative heights in the range $l_2 = 1$, 2, and 3 the nonuniformity of the impurity distribution increases due to broadening of the area occupied by the zone with extremal values of the impurity concentration.

NOTATION

x_0, characteristic dimension; x_i $(i = 1, 2)$, a dimensional coordinate; L_i $(i = 1, 2)$, height and width of the crystallizer cavity; r_i, e_i $(i = 1, 2)$, dimensional coordinates of the phase transition in the Ox_1x_2 coordinate system; T, T_0, and T_K, current temperature, initial temperature, and melt crystallization temperature; ρ, density of the melt; P, P_{max}, and P_{min}, current pressure, maximum pressure, and minimum pressure in the system; c, c_0, current and initial impurity concentration; e_2, unit vector having the same direction as the direction as the force of gravity; $\bar{g}$, acceleration due to gravity; β, coefficient of thermal expansion; γ, diffusion broadening coefficient; $\bar{u}$, velocity of convective motion; ν, kinematic viscosity; k, equilibrium impurity distribution coefficient; t, current time; D, diffusion coefficient; a, thermal diffusivity; $\Delta T = T_0 - T_K$, initial overheating of the melt; $\eta_i = x_i/x_0$ $(i = 1, 2)$, dimensionless coordinate; $l_i = L_i/x_0$ $(i = 1, 2)$ is the relative height and width of the crystallizer cavity in the coordinate system $0\eta_1\eta_2$; $R_i = r_i/x_0$, $\varepsilon_i = e_i/x_0$, dimensionless coordinates of the phase-transition boundary in the $0\eta_1\eta_2$ coordinate system; $\bar{U} = \bar{u}/u_0$, dimensionless velocity of convective motion; $Gr = |\bar{g}|\beta\Delta T x_0^3/\nu^2$, Grashof hydrodynamic number; $Gr_D = |\bar{g}|\gamma c_0 x_0^3/\nu^2$, Grashof diffusion number; $Fo = Dt\ x_0^2$, dimensionless time, $Sm = \nu/D$, Schmidt number; and $Lu = D/a$, Lewis number.

LITERATURE CITED

1. I. O. Kulik and G. E. Zil'berman, "The impurity distribution when a crystal is grown from the melt," in: Crystal Growth [in Russian], Moscow, Vol. 3 (1961), pp. 85-89.
2. P. F. Zavgorodnii, F. V. Nedopekin, and I. L. Povkh, "Hydrodynamics and heat and mass transfer in a hardening melt," Inzh.-Fiz. Zh., 33, No. 5, 922-930 (1977).

3. B. Ya. Lyubov, Theory of Crystallization in Large Volumes [in Russian], Nauka (1975).
4. B. I. Vaiman and E. L. Tarunin, "The effect of crystallization on the process of free convection in melted metals," in: Hydrodynamics [in Russian], Perm, No. 4, 107-118 (1972).
5. É. A. Iodko et al., "Investigation of convective flows in hardening ingots," Izv. Akad. Nauk SSSR, Metally, No. 2, 102-108 (1971).
6. P. F. Zavgorodnii, I. L. Povkh, and G. M. Sevast'yanov, "Intensity of thermal convection as a function of the Grashof members and the hardening kinetics of a melt," Teplofiz. Vys. Temp., 14, No. 4, 823-828 (1976).
7. A. A. Samarskii, Introduction to the Theory of Difference Schemes [in Russian], Nauka, Moscow (1973).
8. N. N. Yanenko, The Method of Fractional Steps in Multivariate Problems of Mathematical Physics [in Russian], Nauka, Novosibirsk (1957).
9. P. F. Zavgorodnii, "Numerical investigation of thermally concentrated convection in the liquid nucleus of a crystallizing binary melt," Inzh.-Fiz. Zh., 35, No. 1, 155-162 (1978).
10. P. F. Zavgorodnii et al., "Calculation of the impurity distribution in a crystallizing ingot," Izv. Vyssh. Uchebn. Zaved., Metall., No. 3, 47-50 (1977).

THERMAL DIFFUSIVITY OF INHOMOGENEOUS SYSTEMS

1. TEMPERATURE-FIELD CALCULATION

G. N. Dul'nev and A. V. Sigalov

UDC 536.24.02

The possibility of analyzing the nonsteady temperature fields of inhomogeneous systems using the quasi-homogeneous-body model is investigated.

Definition of Quasi-Homogeneous Body

A system consisting of homogeneous regions (components) divided by boundary surfaces is usually referred to as inhomogeneous or heterogeneous. Often, in order to calculate the temperature field, this body is replaced by a quasi-homogeneous body with effective thermal conductivity and diffusivity (λ, a) and volume specific heat ($c\rho$). It is then postulated that the temperature field of this body is described at all points by the equation

$$\frac{1}{a}\frac{\partial t}{\partial \tau} = \nabla^2 t, \tag{1}$$

and in specifying the conditions at the external boundaries the effective thermal conductivity is used. This is determined either experimentally, or by the methods of generalized conduction theory [1], and is equal to the ratio of the mean flow $\langle \mathbf{q} \rangle$ through the body and the mean temperature gradient $\langle \nabla t \rangle$ in the body

$$\lambda = -\langle \mathbf{q} \rangle / \langle \nabla t \rangle . \tag{2}$$

The effective volume specific heat is determined from the additive formula

$$c\rho = \sum_{i=1}^{k} c_i \rho_i m_i , \tag{3}$$

and the effective thermal diffusivity is found from a formula valid for a homogeneous body

$$a = \lambda / c\rho. \tag{4}$$

This approaches to the analysis of inhomogeneous-system temperature fields is widely known, but it is not possible to find a sufficiently general justification of this method in the literature. In the present work, the error involved in passing to a quasi-homogeneous body for the calculation of nonsteady temperature fields is investigated, and the limits of applicability of the model in Eqs. (1)-(4) are established.

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 1, pp. 126-133, July, 1980. Original article submitted September 12, 1979.

0022-0841/80/3901-0803$07.50